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SOLUTIONS OF PROBLEMS.

ALGEBRA.

462. Proposed by H. S. UHLER, Yale University.

Show how to transform A into S , where these symbols denote the equivalent formulæ for the general case of Calculus Problem No. 363, pages 52 and 54 in the February, 1916, MONTHLY:

$$A \equiv 4\pi R^2 - 4nR^2 \sin^{-1} \left(\frac{R \sin \pi/n}{\sqrt{R^2 - a^2}} \right) + 2anR \sin^{-1} \left[\frac{2a(\tan \pi/n)(\sqrt{R^2 - a^2 \sec^2 \pi/n})}{R^2 - a^2} \right],$$

$$S \equiv 4nR \left\{ a \sin^{-1} \left(\tan \frac{\pi}{n} \cdot \frac{a}{\sqrt{R^2 - a^2}} \right) - R \sin^{-1} \left[\frac{1}{2} \sin \frac{2\pi}{n} \cdot \frac{R - \sqrt{R^2 - a^2 \sec^2 \pi/n}}{\sqrt{R^2 - a^2}} \right] \right\}.$$

SOLUTION BY GEO. W. HARTWELL, Hamline University.

$$A \equiv 4nR \left\{ R \left[\frac{\pi}{n} - \sin^{-1} \left(\frac{R \sin \pi/n}{\sqrt{R^2 - a^2}} \right) \right] + \frac{1}{2} a \sin^{-1} \left[\frac{2a(\tan \pi/n)(\sqrt{R^2 - a^2 \sec^2 \pi/n})}{R^2 - a^2} \right] \right\} \\ \equiv 4nR(a\gamma + R\beta).$$

Let

$$\sin^{-1} \left(\frac{R \sin \pi/n}{\sqrt{R^2 - a^2}} \right) = \alpha \quad \text{and} \quad \frac{\pi}{n} - \alpha = \beta.$$

Then,

$$\sin \beta = \sin \frac{\pi}{n} \cos \alpha - \cos \frac{\pi}{n} \sin \alpha.$$

$$\cos \alpha = \sqrt{1 - \frac{R^2 \sin^2 \pi/n}{R^2 - a^2}} = \sqrt{\frac{R^2 \cos^2 \pi/n - a^2}{R^2 - a^2}} = \frac{\cos \pi/n \sqrt{R^2 - a^2 \sec^2 \pi/n}}{\sqrt{R^2 - a^2}}.$$

Hence,

$$\sin \beta = \sin \frac{\pi}{n} \cos \frac{\pi}{n} \left[\frac{\sqrt{R^2 - a^2 \sec^2 \pi/n} - R}{\sqrt{R^2 - a^2}} \right],$$

or

$$\beta = -\sin^{-1} \left[\frac{1}{2} \sin \frac{2\pi}{n} \cdot \frac{R - \sqrt{R^2 - a^2 \sec^2 \pi/n}}{\sqrt{R^2 - a^2}} \right].$$

Let

$$\sin^{-1} \frac{2a(\tan \pi/n)(\sqrt{R^2 - a^2 \sec^2 \pi/n})}{R^2 - a^2} = \gamma.$$

Then,

$$\sin \frac{1}{2} \gamma = \sqrt{\frac{1}{2}(1 - \cos \gamma)} = \sqrt{\frac{1}{2} \left(1 - \sqrt{1 - \frac{4a^2 \tan^2 \pi/n (R^2 - a^2 \sec^2 \pi/n)}{(R^2 - a^2)^2}} \right)} \\ = \sqrt{\frac{1}{2} \left(1 - \frac{\sqrt{R^4 - 2a^2 R^2 (1 + 2 \tan^2 \pi/n) + a^4 (1 + 4 \sec^2 \pi/n \tan^2 \pi/n)}}{R^2 - a^2} \right)} \\ = \sqrt{\frac{1}{2} \left(1 - \frac{R^2 - a^2 (1 + 2 \tan^2 \pi/n)}{R^2 - a^2} \right)} = \frac{a \tan \pi/n}{\sqrt{R^2 - a^2}};$$

hence,

$$\frac{1}{2} \gamma = \sin^{-1} \frac{a \tan \pi/n}{\sqrt{R^2 - a^2}}.$$

Making these substitutions, we have $A \equiv S$.

Also solved by the PROPOSER.